PHYSICAL REVIEW E VOLUME 57, NUMBER 5 MAY 1998

Dripping faucet with ants

Eric Bonabeau,[1,*] Guy Theraulaz,[2] Jean-Louis Deneubourg,[3] Arnaud Lioni,[2,3] François Libert,[2] Christian Sauwens,[3]
and Luc Passera[2]
[1]*Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, New Mexico 87501*
[2]*Laboratoire d'Ethologie et de Psychologie Animale, CNRS UMR 5550, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse Cédex, France*
[3]*Unit of Theoretical Behavioural Ecology, Service de Chimie-Physique, CP 231, Université Libre de Bruxelles, Boulevard du triomphe, 1050 Brussels, Belgium*

(Received 21 November 1997)

The formation of droplets of ants is observed under certain experimental conditions. An aggregate forms at the end of a rod, the size of this aggregate fluctuates, and a droplet containing up to 40 ants eventually falls down. When the flux of incoming ants is sufficient, this process can continue for several hours, leading to the formation and fall of tens of droplets. This phenomenon is reminiscent of a leaky faucet, a well-known example of a simple chaotic system. It is found that the similarity is more than apparent: the time series of drop-to-drop intervals appears to result from a nonlinear low-dimensional dynamics, and the interdrop increments exhibit long-range anticorrelations. [S1063-651X(98)05505-6]

PACS number(s): 87.10.+e, 05.40.+j, 05.70.Ln

The dripping faucet was introduced as an early and simple example of a chaotic system [1]. It has been the starting point of several experimental and theoretical investigations [1–4]. The physics of the system involves surface tension, which decreases the dripping rate, and gravity, which acts as a nonlinearly coupled competitive force. Here we report the results of laboratory experiments performed with ants, where ants aggregate at the end of a rod to form droplets that eventually fall down. A lot of the properties of this many-body biological system are similar to those of a leaky faucet: for example, the drop-to-drop interval time series appears to be low dimensional [1,2] and the interdrop increments exhibit long-range anticorrelations [3]. The physical mechanisms underlying this phenomenon, although they are not fully understood, are likely to result from a competition between gravity and the equivalent of tension or cohesive forces among ants.

The formation of bridges or chains in some insect colonies is one of the most striking collective patterns that arises in the animal kingdom. Such structures have been reported, for example, in the army ant *Eciton burchelli* [5], in the African weaver ant *Oecophylla longinoda* [5,6], or in the honeybee *Apis mellifica* [7]: workers form bridges or chains by linking their bodies when they explore a new space, form a bivouac, or cooperate in nest building. How these bridges or chains form is to a large extent an unsolved question. We have carried out experiments with the ant *Linepithema humile* (Mayr) [8], which is not known to form bridges: however, we have observed that these ants can aggregate to form droplets of up to 40 individuals under specific conditions, described below. The formation of these basic structures can bring much insight into the dynamics of bridge or chain formation in ants, where such processes have been observed in natural conditions. In addition, the fact that the dynamic properties of the process of droplet formation are similar to those observed in other physical and biological systems indicates that such properties may be "generic."

The ants can run along a rod, starting in the middle of the nest, and ending in an separate arena at a height of about 10 cm above the ground (Fig. 1): an ant at the end of the rod can either decide to go back to the nest and succeed, or may be stuck in a forming droplet (Fig. 2) and eventually fall to the ground where she is retrieved, and put back into the nest at the end of the experiment. Ants were starved for several days to stimulate exploratory behavior. A colony contains about 10^5 individuals, 20% of which perform outside tasks such as foraging or exploration. Exploration is accompanied with recruitment through trail laying. This leads to the formation of a network of chemical trails through a positive feedback effect. Once a trail leads to the end of the rod, a relatively stable flux of incoming ants is established for up to several hours, after which exploration of the rod ceases. Measurements were analyzed only during the period immediately following the establishment of a stable recruitment. The net flux of ants fluctuates during this period (from nine to 27 ants/5 s, with an average of 19 ants/5 s). The experiment was repeated eight times, producing eight series of up to 300 drops.

Several quantities have been measured during the experiments: the incoming flux of ants at the beginning of the rod, the outgoing flux of ants at the same point, the number of ants located in the vicinity of the rod's end (and therefore likely to fall), the sizes of droplets, and the time intervals separating two successive droplets. In this paper, we focus

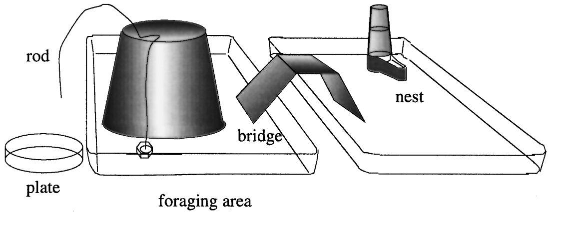

FIG. 1. Experimental setup.

---

*Electronic address: bonabeau@santafe.edu

© 1998 The American Physical Society

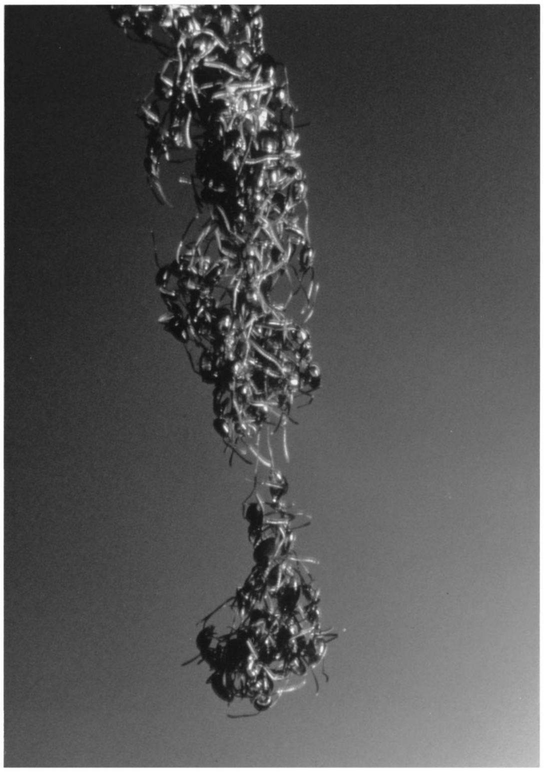

FIG. 2. A droplet is about to fall, while the next droplet is already forming at the end of the rod.

on the last two measurements, which allow us to draw a clear parallel with a dripping faucet.

Figures 3(a) and 3(b) show the reconstructed phase portraits in three dimensions of two series: $\Delta t(n)$, the time interval separating the $(n+1)$th droplet from the nth droplet, and $s(n)$, the size of the nth droplet. In both cases, the ''time'' lag of the reconstruction is 1. It can already be seen that the $\Delta t(n)$ series has more structure than the $s(n)$ series. We use Takens maximum-likelihood method [9] to evaluate the correlation dimension D_2 of the experimental series. Two surrogate data methods (phase randomized and Gaussian scaled surrogates) [10] were used. The value of D_2 found in the $\Delta t(n)$ series rapidly saturates with increasing embedding dimension at a value $D_2 = 1.7 \pm 0.04$, indicating that the underlying dynamics may be low dimensional (in their model of a dripping faucet, Sanchez-Ortiz and Salas-Brito [4] find $D_2 = 1.25 \pm 0.08$ for their electrocardiogram attractor); longer series would be desirable, but could not be obtained in the experiments. Surrogate data series, exhibiting a significantly different behavior (t test, $p < 0.01$ for both surrogate data sets), support the hypothesis that the dynamics is nonlinear. Aggregation at the end of the rod is a many-body phenomenon, that involves a large number of degrees of freedom: our result suggests that these degrees of freedom are to a large extent interdependent, and that the dynamical process involves only a few effective variables. The result is very different for the $s(n)$ time series, where no evidence of low-dimensional nonlinear dynamics is found: D_2 increases with increasing embedding dimension, and so do the surrogate series.

How can the series of interdrop intervals be nonlinear low dimensional, while that of droplet sizes is not? Both series can be considered as sequences of snapshots of events which do not take place on the same time scale. Videotape analysis of droplets shows that very rapid and complex movements occur within droplets, at a time scale which is short compared with the time interval separating two successive droplets: the number of ants in a given droplet is the result of a process involving fast interacting variables, which weakly depend on the slower process of aggregation at the end of the rod. Although it is not completely clear from the data, the factors that determine when a droplet falls certainly include the fact that the set of links between the surface of the rod and the rest of the droplet is pushed beyond ''capacity;'' this notion of capacity must be clarified, since it may involve several factors, such as the ''history'' of a link, the exhausting movements of other ants in the droplet, or even a saturation, which does not depend on the droplet's mass but on the physiological state or the lack of ''motivation'' of the various links. Assuming that the $\Delta t(n)$ series is deterministic, let F be the deterministic application such that $\Delta t(n+1) = F[\Delta t(n)]$. If, due to rapid fluctuations, $\Delta t(n)$ is transformed into $\Delta t(n) + \delta^{(n)}$, where $\delta^{(n)}$ is a small noise term, the modified trajectory $[\ldots, \Delta t(n) + \delta^{(n)}, F(\Delta t(n) + \delta^{(n)}) + \delta^{(n+1)}, \ldots]$ is not unlikely to remain a trajectory of

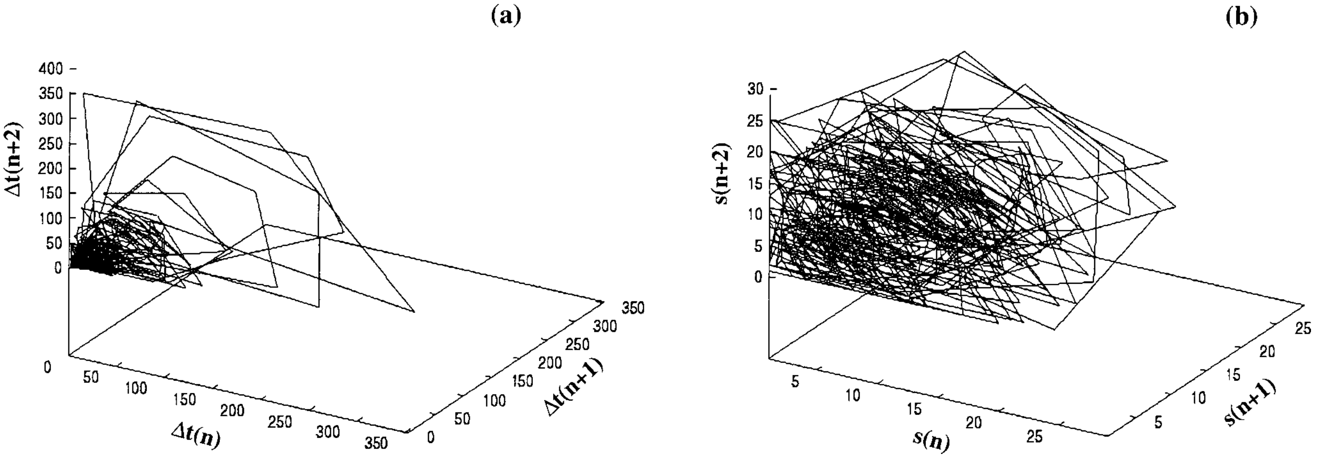

FIG. 3. (a) Reconstructed phase portrait $[\Delta t(n), \Delta t(n+1), \Delta t(n+2)]$ of the $\Delta t(n)$ series. (b) Reconstructed phase portrait $[s(n), s(n+1), s(n+2)]$ of the $s(n)$ series.

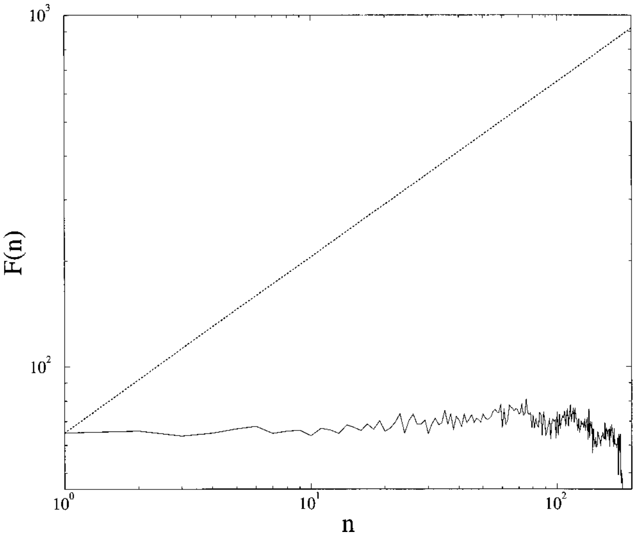

FIG. 4. Fluctuations $F(n)$ of the interdrop increment $I(n)$. A random walk would have produced the dotted line ($\alpha=0.5$).

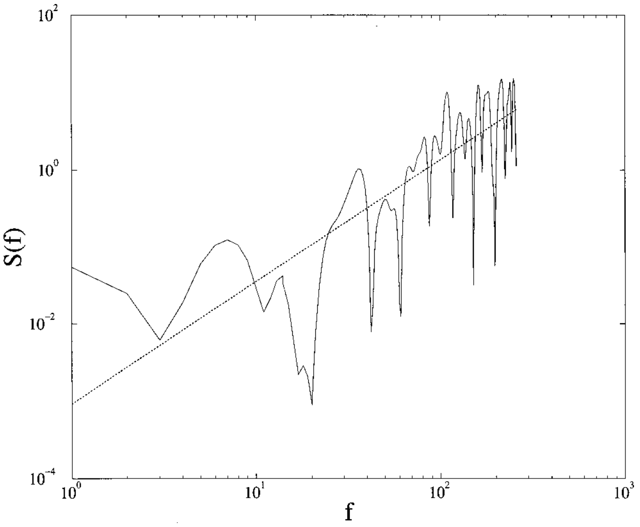

FIG. 5. Power spectrum of the interdrop increment $I(n)$. The dotted line represents the best power-law fit $S(f)\propto f^{\beta}$, $\beta=1.58$.

the deterministic system if ''shadowing'' applies [11]. On the other hand, $s(n)$ will be drastically modified by $\delta^{(n)}$ if it fluctuates on a time scale shorter than $\delta^{(n)}$ [$s(n)$ is also likely to be dependent on fluctuations of the flux of ants]. In effect, $s(n)$ is the size of the droplet at the exact time when it falls. Let $s(n)=s(t_n\equiv\Sigma_{i=1}^{n}\Delta t(i))$. The values of $s(t_n)$ and $s(t_n+\delta^{(n)})$ may greatly differ, even for a small $\delta^{(n)}$, because a large number of reorganizing events took place in the droplet between t_n and $t_n+\delta^{(n)}$.

Finally, we analyzed the series of interdrop increments $I(n)=\Delta t(n+1)-\Delta t(n)$. Let us define the mean fluctuation function $F(n)=\langle|\Delta t(n'+n)-\Delta t(n)|\rangle_{n'}$, where $\langle\cdots\rangle_{n'}$ denotes averaging over n': $F(n)$ quantifies the magnitude of the fluctuations over different scales n [3,12]. We obtain $F(n)\propto n^{\alpha}$ (Fig. 4), with $\alpha=-0.05$, whereas a random walk would have produced $\alpha=0.5$ (dotted line in Fig. 4). Similarly, the probability density $P(I)$ of $I(n)$ seems to be well described by a Lévy law [$P(I)=\pi^{-1}\int_0^{\infty}\exp(-aq^{\Psi})\cos(qI)dq$, where $\psi\approx1.5$; a Lorentzian distribution would correspond to $\psi=1$, and a Gaussian distribution to $\psi=2$] [3,12], but more data would be needed to confirm this observation. In order to study the dynamical properties of $I(n)$ further, the power spectrum of $I(n)$, $S(f)=N^{-1}\langle|\Sigma_{n=1}^{N}I(n)e^{2i\pi ft}|^2\rangle$, where $\langle\cdots\rangle$ denotes averaging over the eight time series, has been computed. It is found that $S(f)\propto f^{\beta}$, with $\beta=1.58$ (Fig. 5) [one would expect $\beta=1-2\alpha=1.1$: this discrepancy may result from the fact that neither $S(f)$ nor $F(n)$ are perfect power laws], indicating that $I(n)$ is anticorrelated [13]: interdrop increments are ordered in a particular way, where positive and negative values of $I(n)$ are likely to alternate in time. The origin of such anticorrelations can be traced back to the dynamics of droplets: a rapid succession of two droplets is likely to be followed by a longer time interval, because the aggregate of ants at the end of the rod has been depleted as a result of the two droplets that just fell down, and it takes time for a new droplet to form again and fall. Peng *et al.* [12] found similar long-range anticorrelations and non-Gaussian behavior in heartbeat-to-heartbeat intervals of healthy subjects. Penna *et al.* [3] also found the same type of behavior in a model leaky faucet, suggesting that the analogy between the dripping faucet and our experiment is not only superficial, and that similar sets of competing forces (cohesive forces and gravity) play similar roles in determining the dynamics of the system.

Although we have no ''functional'' explanation for this phenomenon *in this species* of ants—it may be a nonadaptive side effect of functional traits, such as recruitment, used in exploratory and foraging activities, and may possibly occur only in ''pathological'' situations—it is nevertheless important to point out the similarities that exist between its dynamic properties and those of other physical or biological systems.

E.B. would like to thank Interval Research for financial support at the Santa Fe Institute. E.B. and G.T. acknowledge support from the GIS (Groupe d'Intérêt Scientifique) Sciences de la Cognition. The correlation dimension analysis has been performed in part with Ts tools, courtesy of J. Theiler.

---

[1] R. S. Shaw, *The Dripping Faucet as a Model Chaotic System* (Aerial, Santa Cruz, 1984).

[2] P. Martien, S. C. Pope, P. L. Scott, and R. S. Shaw, Phys. Lett. A **110**, 399 (1985); J. Austin, *ibid.* **155**, 148 (1991); J. C. Sartorelli, W. M. Gonçalves, and R. D. Pinto, Phys. Rev. E **49**, 3963 (1994).

[3] T. J. P. Penna, P. M. C. de Oliveira, J. C. Sartorelli, W. M. Gonçalves, and R. D. Pinto, Phys. Rev. E **52**, R2168 (1995).

[4] G. I. Sanchez-Ortiz and A. L. Salas-Brito, Phys. Lett. A **203**, 300 (1995).

[5] B. Hölldobler and E. O. Wilson, *The Ants* (Springer-Verlag, Berlin, 1990).

[6] B. Hölldobler and E. O. Wilson, Sci. Am. **146**, 237 (1977).

[7] R. Darchen, Insectes Sociaux **9**, 103 (1962).

[8] S. H. Skaife, Trans. R. Soc. South Africa **34**, 355 (1955).

[9] F. Takens, Lect. Notes Math. **1125**, 99 (1984); J. Theiler, Phys. Rev. A **41**, 3038 (1990).

[10] J. Theiler, S. Eubank, A. Longtin, B. Galdrikian, and J. D. Farmer, Physica D **58**, 77 (1992); N. Pradhan and P. K. Sadavisan, Int. J. Bifurcation Chaos Appl. Sci. Eng. **7**, 173 (1997).

[11] C. Grebogi, S. M. Hammel, J. A. Yorke, and T. Sauer, Phys. Rev. Lett. **65**, 1527 (1990).

[12] C.-K. Peng, J. Mietus, J. M. Hausdorff, S. Havlin, H. E. Stanley, and A. L. Goldberger, Phys. Rev. Lett. **70**, 1343 (1993).

[13] S. Havlin, R. B. Selinger, M. Schwartz, H. E. Stanley, and A. Bunde, Phys. Rev. Lett. **61**, 1438 (1988).